The Quadratic Formula & the Discriminant

Goals of Lesson:
• Apply the quadratic formula to determine the solutions to a quadratic equation (or x-intercepts)
• Use the discriminant to determine the nature and quantity of the solutions to a quadratic equation

DoDEA Mathematics Standards for Algebra II Addressed:
• A2.1.4 Determine rational and complex zeros for quadratic equations
• A2.5.6 Describe the characteristics of a quadratic function (maximum, minimum, zero values, y-intercepts) and use them to solve real world problems (use technology where appropriate):

Purpose of the Quadratic Formula

The quadratic formula can be used to:
• Determine the ________________ to a quadratic equation.
• Determine the ________________ of the graph of a quadratic function.

What is the Quadratic Formula?

The solution to an equation of the form _________________________, where a, b, and c are real numbers and a ≠ __:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Things to Remember when Using the Quadratic Formula

• The equation must be equal to ________
• Identify a, b, and c in the equation is equal to 0
• Substitute values into the ____________ (write it down - even if you are going to use a calculator)
• Follow ________________ ~ if you are using a calculator, put _____________ around negative numbers
Guided Practice

Directions: Solve each equation by using the Quadratic Formula.

Ex 1) \( m^2 - 5m = 7 \)
Ex 2) \( 2 + n - 3n^2 = 4 - n \)

The Discriminant

What is the purpose of the discriminant?

- Lets you know what ______ of solutions a quadratic equation will have - ________ or __________:
- Lets you know _________ solutions a quadratic equation will have - _____ or _____
- With respect to the __________ of a quadratic function, lets you know if it will have _____, _____, or _____ x-intercepts

What is the Discriminant? How is it interpreted?

The discriminant is the expression ______________. It comes from the __________ __________.

Given \( ax^2 + bx + c = 0 \):

If \( b^2 - 4ac > 0 \), then there are ________________.

If \( b^2 - 4ac = 0 \), then there is ____________________.

If \( b^2 - 4ac < 0 \), then there are ________________.
Guided Practice

Directions: Describe the number and type of solutions of the given quadratic equation.

Ex 1) \(x^2 + 10x + 23 = 0\)  
Ex 2) \(x^2 - 5x = 5x^2 - 3x + 1\)

Finding \(x\)-Intercepts Using a Graphing Calculator

\[ f(x) = 3x^2 - 4x - 2 \]

1. Graph the equation in Y= menu. Set the window so that the \(x\)-intercepts are visible (Using ZOOM OUT can help check that all such points on the graph are present.)

2. To find the point(s) where the graph crosses the \(x\)-axis and the function has zeros, use 2nd CALC, 2 (ZERO).

3. Answer the question "left bound?" with ENTER after moving the cursor close to and to the left of an \(x\)-intercept. Answer the question "right bound?" with ENTER after moving the cursor close to and to the right of this \(x\)-intercept.

4. To the question "guess?" press ENTER. The coordinates of the \(x\)-intercept will be displayed. Repeat to get the other \(x\)-intercept.
6.5 Practice

Find the discriminant of each quadratic equation then state the number and type of solutions.

1) $8x^2 + 2x + 8 = 0$
2) $-n^2 - 4 = 4n$

3) $x^2 + 5 = 4x$
4) $-5b^2 - 8 = -10b$

Solve each equation with the quadratic formula.

5) $8n^2 + 4n - 17 = 0$
6) $m^2 + 8 = 0$

7) $6x^2 + 9x - 4 = 3$
8) $8x^2 - 5x - 14 = -8$

9) $2x^2 - 4 = -3x$
10) $x^2 = -9 - 3x$

11) $11a^2 = 3$
12) $5b^2 - 9b - 13 = -11b$

Use a graphing calculator to determine the x-intercepts of each quadratic function. If necessary, round your answers to the nearest hundredths place.

13) $y = -2x^2 + 10x - 10$
14) $y = 3x^2 + 21x + 32$
Interceptions

In a football game, a defensive player jumps up to block a pass by the opposing team’s quarterback. The player bates the ball downward with his hand at an initial vertical velocity of -50 feet per second when the ball is 7 feet above the ground. How long do the defensive player’s teammates have to intercept the ball before it hits the ground?

The model for an object that is launched is $h(t) = -16t^2 + v_0t + h_0$.

Mortar Fire

During World War I, mortars were fired from trenches 3 feet down. The mortars had a velocity of 150 ft/s. Determine how long it would take for the mortar shells to strike their targets.

Bridge Problem - Determining the Equation of a Quadratic Function

The cables supporting a straight-line suspension bridge are nearly parabolic in shape. Suppose that a suspension bridge is being designed with concrete supports 160 ft apart and with vertical cables 30 ft above road level at the midpoint of the bridge and 80 ft above the road level at a point 50 feet from the midpoint for the bridge.

a) Find the quadratic function that represents the curve cable.

b) What are the heights of the concrete supports?